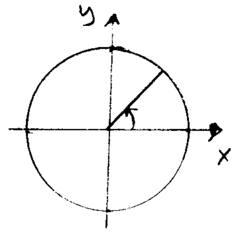
Proppen
060822
Kap 4 [4.2]

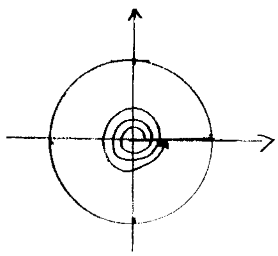
Ange i såväl varv som grader och illustrera grafiskt en vridning av

a) $\frac{\pi}{4}$ radianer



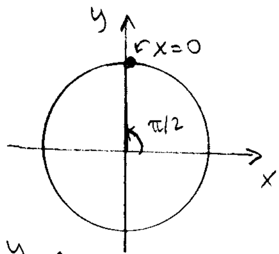
c) 6π radianer $= 2\pi + 2\pi + 2\pi$ rad $= 3 \cdot 2\pi$ radianer

3 varv

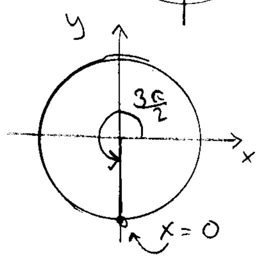


[4.3] Beräkna

a) $\cos \frac{\pi}{2} = 0$



f) $\cos \frac{3\pi}{2} = 0$



[4.4] Beräkna

a) $\sin 27\pi = \sin(\pi + 26\pi) = \sin(\pi + 13 \cdot 2\pi) = \sin \pi = 0$

b) $\cos 1016\pi = \cos(0 + 508 \cdot 2\pi) = \cos 0 = 1$

[4.5] Beräkna

a) $\cos \frac{17\pi}{4} = \cos\left(\frac{\pi}{4} + \frac{16\pi}{4}\right) = \cos\left(\frac{\pi}{4} + 4\pi\right) = \cos\left(\frac{\pi}{4} + 2 \cdot 2\pi\right)$

$= \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

b) $\cos\left(-\frac{7\pi}{4}\right) = \cos\left(\frac{\pi}{4} - \frac{8\pi}{4}\right) = \cos\left(\frac{\pi}{4} - 2\pi\right) = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

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Lös ekvationen

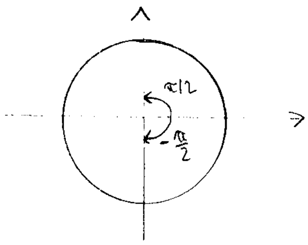
Kap4

[4,14] a) $\cos x = \cos\frac{\pi}{6} = \cos(\frac{\pi}{6} + n\cdot 2\pi)$

$x = \frac{\pi}{6} + n\cdot 2\pi$ eller $x = -\frac{\pi}{6} + n\cdot 2\pi$

c) $\cos x = 0 \Rightarrow \cos x = \cos\frac{\pi}{2}$

$x = \frac{\pi}{2} + n\cdot 2\pi$ eller $x = -\frac{\pi}{2} + n\cdot 2\pi \Rightarrow x = \frac{\pi}{2} + n\cdot\pi$



d) $\cos x = \frac{1}{2} \Rightarrow \cos x = \cos\frac{\pi}{3} = \cos(\frac{\pi}{3} + n\cdot 2\pi)$

$x = \frac{\pi}{3} + n\cdot 2\pi$ eller $x = -\frac{\pi}{3} + n\cdot 2\pi$

[4.15] Lös ekvationen

a) $\sin x = \sin\frac{\pi}{5} \Rightarrow \sin x = \sin(\frac{\pi}{5} + n\cdot 2\pi)$

$\underline{x = \frac{\pi}{5} + n\cdot 2\pi}$ eller $x = \pi - \frac{\pi}{5} + n\cdot 2\pi$

$\underline{x = \frac{4\pi}{5} + n\cdot 2\pi}$

b) $\sin x = \frac{\sqrt{3}}{2}$ $\sin x = \sin\frac{\pi}{3} = \sin(\frac{\pi}{3} + n\cdot 2\pi)$

$\underline{x = \frac{\pi}{3} + n\cdot 2\pi}$ eller $x = \pi - \frac{\pi}{3} + n\cdot 2\pi$

$\underline{x = \frac{2\pi}{3} + n\cdot 2\pi}$

c) $\sin x = 0 \Rightarrow \sin x = \sin 0 = \sin(0 + n\cdot 2\pi) = \sin(n\cdot 2\pi)$

$x = n\cdot 2\pi$ eller $x = \pi + n\cdot 2\pi$

$\Rightarrow \underline{x = n\cdot\pi}$

$\Rightarrow$

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Visa att för alla v gäller

[4.25]

a) $\cos 2v = 1 - 2\sin^2 v$

$VL = \cos 2v = \left\{ \begin{matrix} \text{Formeln för} \\ \text{dubbla vinkeln} \end{matrix} \right\} = \cos^2 v - \sin^2 v = \left\{ \begin{matrix} \text{Trigonometriska} \\ \text{ettan} \end{matrix} \right\}$

$= 1 - \sin^2 v - \sin^2 v = 1 - \sin^2 v = HL$ ok!

c) $\cos 3v = 4\cos^3 v - 3\cos v$

$VL = \cos 3v = \cos(2v + v) = \cos 2v \cdot \cos v - \sin 2v \cdot \sin v$

$= (\cos^2 v - \sin^2 v)\cos v - (2\cos v \cdot \sin v)\sin v$

$= \cos^3 v - \cos v \cdot \sin^2 v - 2\cos v \cdot \sin^2 v$

$= \cos^3 v - 3\cos v \sin^2 v = \cos^3 v - 3\cos v\,(1 - \cos^2 v)$

$= \cos^3 v - 3\cos v + 3\cos^3 v = 4\cos^3 v - 3\cos v = HL$ ok!

[4.26] Beräkna m ha kända värden och formlerna (8) eller (9), de exakta värdena av $\cos\frac{\pi}{8}$, $\sin\frac{\pi}{8}$ och $\cos\frac{\pi}{16}$

$\cos^2 v = \frac{1}{2}(1 + \cos 2v) \Rightarrow \cos v = \pm\sqrt{\frac{1 + \cos 2v}{2}}$

$\cos\frac{\pi}{8} = \pm\sqrt{\frac{1 + \cos\left(\frac{2\pi}{8}\right)}{2}} = \pm\sqrt{\frac{1 + \cos\frac{\pi}{4}}{2}} = \pm\sqrt{\frac{1 + \frac{1}{\sqrt{2}}}{2}}$

$= \pm\sqrt{\frac{1}{2} + \frac{1}{2\sqrt{2}}} = \pm\sqrt{\frac{1}{2} + \frac{\sqrt{2}}{4}} = \pm\sqrt{\frac{2 + \sqrt{2}}{4}} = \underline{\pm\frac{1}{2}\sqrt{2 + \sqrt{2}}}$

$\sin^2 v = \frac{1}{2}(1 - \cos 2v) \Rightarrow \sin v = \pm\sqrt{\frac{1 - \cos 2v}{2}}$

$\sin\frac{\pi}{8} = \pm\sqrt{\frac{1 - \cos\frac{\pi}{4}}{2}} = \pm\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{2}}} = \underline{\pm\frac{1}{2}\sqrt{2 - \sqrt{2}}}$

$\cos\frac{\pi}{16} = \pm\sqrt{\frac{1 + \cos\frac{\pi}{8}}{2}} = \pm\frac{1}{2}\sqrt{2 + 2\cos\frac{\pi}{8}} = \pm\frac{1}{2}\sqrt{2 \pm 2 \cdot \frac{1}{2}\sqrt{2 + \sqrt{2}}}$

$= \underline{\pm\frac{1}{2}\sqrt{2 \pm \sqrt{2 + \sqrt{2}}}}$

$\Rightarrow$

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Lös ekvationen

Kap 4

[4.29] a) $\sin^2 x = \frac{3}{4}$

$\sin x = \pm\frac{\sqrt{3}}{2}$ dvs 1) $\sin x = \frac{\sqrt{3}}{2}$ eller 2) $\sin x = -\frac{\sqrt{3}}{2}$

1) $\sin x = \frac{\sqrt{3}}{2} \Rightarrow \sin x = \sin\frac{\pi}{3} = \sin\left(\frac{\pi}{3} + n \cdot 2\pi\right)$

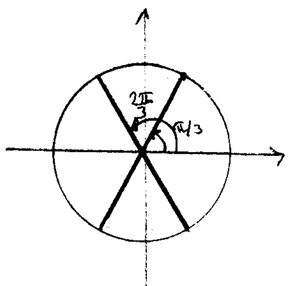
$\underline{x = \frac{\pi}{3} + n \cdot 2\pi}$ eller $x = \pi - \frac{\pi}{3} + n \cdot 2\pi$

$\underline{x = \frac{2\pi}{3} + n \cdot 2\pi}$

2) $\sin x = -\frac{\sqrt{3}}{2} \Rightarrow \sin x = \sin\left(-\frac{\pi}{3}\right) = \sin\left(-\frac{\pi}{3} + n \cdot 2\pi\right)$

$\underline{x = -\frac{\pi}{3} + n \cdot 2\pi}$ eller $x = \pi - \left(-\frac{\pi}{3}\right) + n \cdot 2\pi$

$\underline{x = \frac{4\pi}{3} + n \cdot 2\pi}$



$x = \frac{\pi}{3} + n \cdot \pi$

$x = \frac{2\pi}{3} + n \cdot 2\pi$

[4.30] Lös ekvationen

a) $\sin^2 x + \cos x = \frac{5}{4} \Rightarrow 1 - \cos^2 x + \cos x = \frac{5}{4}$

$\Rightarrow \cos^2 x - \cos x + \frac{1}{4} = 0$ sätt $t = \cos x$

$t^2 - t + \frac{1}{4} = 0 \Rightarrow t_{1,2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{4}} = \frac{1}{2}$

$\cos x = \frac{1}{2} \Rightarrow \cos x = \cos\frac{\pi}{3} = \cos\left(\frac{\pi}{3} + n \cdot 2\pi\right)$

$x = \pm\frac{\pi}{3} + n \cdot 2\pi$

$\Longrightarrow$