Proppen
060811

Mot bättre vetande i matematik

(Lösta uppgifter till rekommenderade uppgifter.)

Kap 1

[1.7] Beräkna.

a) $\frac{1}{2}+\frac{2}{5}-\frac{1}{3}=\frac{1}{2}+\frac{2\cdot3-5\cdot1}{5\cdot3}=\frac{1}{2}+\frac{6-5}{15}=\frac{1}{2}+\frac{1}{15}=\frac{15+2}{2\cdot15}=\frac{17}{30}$

c) $3+\frac{1}{7}-\frac{15}{14}-\frac{1}{2}=\frac{3\cdot2}{2}+\frac{2}{7\cdot2}-\frac{15}{14}-\frac{1}{2}=\frac{3\cdot2}{2}-\frac{1}{2}+\frac{2}{7\cdot2}-\frac{15}{14}$

$=\frac{6}{2}-\frac{1}{2}+\frac{2}{14}-\frac{15}{14}=\frac{6-1}{2}+\frac{2-15}{14}=\frac{5}{2}-\frac{13}{14}$

$=\frac{5\cdot7}{2\cdot7}-\frac{13}{14}=\frac{35-13}{14}=\frac{22}{14}=\frac{11}{7}$

Svar: a) $\frac{17}{30}$

c) $\frac{11}{7}$

[1.8] Beräkna:

a) $\frac{3}{\frac{4}{5}+1}=\frac{3}{\frac{4}{5}+\frac{5}{5}}=\frac{3}{\frac{4+5}{5}}=\frac{3}{\frac{9}{5}}=\frac{3\cdot5}{9}=\frac{5}{3}$

Svar: $\frac{5}{3}$

[1.11] Beräkna

a) $(-2)^3+(-1)^4-(-1)^2=(-2)^2(-2)+(-1)^2(-1)^2-(-1)^2=4\cdot(-2)+1\cdot1-1$

$=-8+1-1=-8$

e) $\left(\frac{5\cdot10^5}{125}\right)^3=\left(\frac{5\cdot10^5}{5^3}\right)^3=\left(\frac{5\cdot(2\cdot5)^5}{5^3}\right)^3=\left(\frac{5\cdot2^5\cdot5^5}{5^3}\right)^3=\left(2^5\cdot5^3\right)^3$

$=\left(2^2\cdot2^3\cdot5^3\right)^3=\left(2^2\cdot(2\cdot5)^3\right)^3=\left(2^2\cdot10^3\right)^3$

$=2^6\cdot10^9=64\cdot10^9$

g) $\frac{9^4\cdot3^{-3}}{(27)^2}=\frac{9^4}{3^3(3\cdot9)^2}=\frac{9^4}{3^3\cdot3^2\cdot9^2}=\frac{9^2}{3^5}=\frac{(3\cdot3)^2}{3^5}=\frac{3^4}{3^5}=\frac{1}{3}$

Svar: a) -8

e) $64\cdot10^9$

g) $\frac{1}{3}$

[1.14] Beräkna

a) $(2^3)^2 - 2^{3^2} = 8^2 - 2^9 = 8^2 - 2^3 \cdot 2^3 \cdot 2^3 = 8^2 - 8^3 = 64 - 512 = \underline{-448}$

[1.15] Multiplicera ihop följande parenteser.

a) $(a+b)(a-c) = a^2 - ac + ab - bc$

b) $(4+q)(4-q) = \{\text{konjugatregeln}\} = 16 - q^2$

d) $(3+t)(7-t) = 21 - 3t + 7t - t^2 = 21 + 4t - t^2$

f) $(2c+5d)^2 = \{\text{kvadreringsregeln}\} = 4c^2 + 20cd + 25d^2$

[1.17] Förenkla

b) $(p+q)(p-q) - (p-q)^2 = \left\{\begin{matrix}\text{Konjugatregeln och} \\ \text{Kvadreringsregeln}\end{matrix}\right\} = p^2 - q^2 - (p^2 - 2pq + q^2)$

$= p^2 - q^2 - p^2 + 2pq - q^2 = 2pq - 2q^2$

c) $(r+s)^2 - (r-s)^2 = r^2 + 2rs + s^2 - (r^2 - 2rs + s^2) = r^2 + 2rs + s^2 - r^2 + 2rs - s^2$
$= 4rs$

[1.24] Dela upp följande uttryck i faktorer, så långt som möjligt

a) $9a^4 - 16b^4 = 3^2(a^2)^2 - 4^2(b^2)^2 \overset{\{\text{omvända konjugatregeln}\}}{=} (3a^2 + 4b^2)(3a^2 - 4b^2)$

$= (3a^2 + 4b^2)((\sqrt{3})^2 \cdot a^2 - 2^2 \cdot b^2) = \{\text{omvända konjugatregeln}\}$

$= (3a^2 + 4b^2)(\sqrt{3}a - 2b)(\sqrt{3}a + 2b)$

Jämför

c) $a(a+b) - (b+1) = a^2 + ab - b - 1 = a^2 + ab + \overbrace{a - a}^{\text{påverkar inte uttrycket}} - b - 1$

$= a(a+b+1) - (a+b+1) = (a-1)(a+b+1)$

Proppen
060816

Förenkla

[1.26]

a) $$\frac{3x+2}{5} - \frac{2x-1}{5} = \frac{3x+2-(2x-1)}{5} = \frac{3x+2-2x+1}{5}$$

$$= \frac{x+3}{5}$$

c) $$\frac{4x-7y}{9y} - \left(\frac{5x+2y}{12y} - 1\right) = \frac{4x-7y}{9y} - \left(\frac{5x+2y}{12y} - \frac{12y}{12y}\right)$$

$$= \frac{4x-7y}{9y} - \left(\frac{5x+2y-12y}{12y}\right) = \frac{4x-7y}{9y} - \left(\frac{5x-10y}{12y}\right)$$

$$= \frac{4x-7y}{9y} + \frac{-5x+10y}{12y} = \left\{\begin{array}{l}\text{Minsta gemensamma} \\ \text{nämnare är } 36y\end{array}\right\}$$

$$= \frac{4(4x-7y)}{36y} + \frac{3(-5x+10y)}{36y} = \frac{16x-28y-15x+30y}{36y}$$

$$= \frac{x+2y}{36y}$$

d) $$2 - \frac{2}{x-1} = \left\{\begin{array}{l}\text{MGN är} \\ x-1\end{array}\right\} = \frac{2(x-1)}{x-1} - \frac{2}{x-1} = \frac{2x-2-2}{x-1}$$

$$= \frac{2x-4}{x-1}$$

[1.27] Förenkla

a) $$\frac{a^6}{b^2} \cdot \frac{b^4}{c^6} \cdot \frac{c^3}{a^2} = \frac{a^6}{a^2} \cdot \frac{b^4}{b^2} \cdot \frac{c^3}{c^6} = a^4 \cdot b^2 \cdot \frac{1}{c^3} = \frac{a^4 \cdot b^2}{c^3}$$

f) $$\frac{\frac{5}{3}x + \frac{5}{6}}{\frac{9}{4}x - \frac{7}{12}} = \left\{\begin{array}{l}\text{MGN är} \\ 12\end{array}\right\} = \frac{\frac{5\cdot4}{12}x + \frac{5\cdot2}{12}}{\frac{9\cdot3}{12}x - \frac{7}{12}} = \frac{\frac{20x+10}{12}}{\frac{27x-7}{12}} = \frac{20x+10}{\cancel{12}} \cdot \frac{\cancel{12}}{27x-7}$$

$$= \frac{20x+10}{27x-7}$$

[1.29] Förenkla (svara exakt)

c) $(\sqrt{12}-\sqrt{3})^2 = 12 - 2\sqrt{12}\cdot\sqrt{3} + 3 = 15 - 2\sqrt{12\cdot 3} = 15 - 2\sqrt{36} = 15 - 2\cdot 6 = 3$

[1.30] Skriv om så att nämnaren inte innehåller rottecken.

c) $\dfrac{1}{1-(\sqrt{3}-1)^2} = \dfrac{1}{1-(3-2\sqrt{3}+1)} = \dfrac{1}{1-(4-2\sqrt{3})} = \dfrac{1}{-3+2\sqrt{3}}$

$=$ [förlänger med $-3-2\sqrt{3}$ på täljare och nämnare] $= \dfrac{1\cdot(-3-2\sqrt{3})}{(-3+2\sqrt{3})\cdot(-3-2\sqrt{3})}$

$= \dfrac{-3-2\sqrt{3}}{9-4\cdot 3} = \dfrac{-3-2\sqrt{3}}{-3} = \dfrac{3+2\sqrt{3}}{3} = 1 + \dfrac{2\sqrt{3}}{3}$

[1.34] Lös ekvationen

a) $2x - \frac{2}{5} = 4 - \frac{7x}{10}$

$2x + \frac{7x}{10} = 4 + \frac{2}{5}$

$10\cdot 2x + \frac{10\cdot 7x}{10} = 4\cdot 10 + \frac{2\cdot 10}{5}$

$20x + 7x = 40 + 4$

$27x = 44$

$x = \frac{44}{27}$

b) $\frac{2x}{7} = \frac{3}{5}$

$\frac{5\cdot 2x}{7} = \frac{5\cdot 3}{5}$

$\frac{7\cdot 5\cdot 2x}{7} = \frac{7\cdot 5\cdot 3}{5}$

$5\cdot 2x = 7\cdot 3$

$10x = 21$

$x = \frac{21}{10}$

Proppen
060517

Lös ekvationen

[1.35]

a) $8(2x-3)(7x+11)=0$ Multiplicera med $\frac{1}{8}$

$(2x-3)(7x+11)=0$

$2x-3=0$ eller $7x+11=0$

$x=\frac{3}{2}$ eller $x=\frac{-11}{7}$

b) $15x^2=16x$

$15x^2-16x=0$

$x(15x-16)=0$

$x=0$ eller $x=\frac{16}{15}$

[1.36] Skriv om följande uttryck som en jämn kvadrat plus eller minus en konstant term, dvs kvadratkomplettera.

a) $x^2+7x+12=x^2+2\cdot\frac{7x}{2}+\left(\frac{7}{2}\right)^2-\left(\frac{7}{2}\right)^2+12=\left(x+\frac{7}{2}\right)^2-\frac{49}{4}+\frac{12\cdot4}{4}$

$=\left(x+\frac{7}{2}\right)^2+\frac{-49+48}{4}=\left(x+\frac{7}{2}\right)^2-\frac{1}{4}$

b) $x^2-9x+22=x^2-2\cdot\frac{9x}{2}+\left(\frac{9}{2}\right)^2-\left(\frac{9}{2}\right)^2+22=\left(x-\frac{9}{2}\right)^2-\frac{81}{4}+\frac{22\cdot4}{4}$

$=\left(x-\frac{9}{2}\right)^2+\frac{-81+88}{4}=\left(x-\frac{9}{2}\right)^2+\frac{7}{4}$

f) $3x^2-24x+40=3\left(x^2-\frac{24x}{3}+\frac{40}{3}\right)=3\left(x^2-8x+\frac{40}{3}\right)$

$=3\left(x^2-2\cdot4x+4^2-4^2+\frac{40}{3}\right)=3\left((x-4)^2-16+\frac{40}{3}\right)$

$=3\left((x-4)^2-\frac{8}{3}\right)=3(x-4)^2-8$

[1.37] Lös ut I ur sambandet $x=\frac{WL^3}{EI}$

multiplicera med $\frac{I}{x}$ i vänster och högerled

$\frac{I}{x}\cdot x=\frac{WL^3}{EI}\cdot\frac{I}{x}$

$I=\frac{WL^3}{Ex}$

[1.48] Lös ut C, $C>0$ ur formeln $Z = \sqrt{Z^2 + (WL - \frac{1}{WC})^2}$

$$Z = \sqrt{R^2 + (WL - \frac{1}{WC})^2}$$

$$Z^2 = R^2 + (WL - \frac{1}{WC})^2$$

$$Z^2 - R^2 = (WL - \frac{1}{WC})^2$$

$$\pm\sqrt{Z^2 - R^2} = WL - \frac{1}{WC}$$

$$\frac{1}{WC} = WL \mp \sqrt{Z^2 - R^2}$$

$$C = \frac{1}{W(WL \mp \sqrt{Z^2 - R^2})}$$

[1.51] Man har $g(x) = 3x^2 + 1$. Bestäm

a) $g(2x)$, $g(x-1)$, $g(x^2)$ och $(g(x))^2$

$$g(2x) = 3(2x)^2 + 1 = 3 \cdot 4x^2 + 1 = 12x^2 + 1$$

$$g(x-1) = 3(x-1)^2 + 1 = 3(x^2 - 2x + 1) + 1 = 3x^2 - 6x + 3 + 1 = 3x^2 - 6x + 4$$

$$g(x^2) = 3(x^2)^2 + 1 = 3x^2 \cdot x^2 + 1 = 3x^4 + 1 = 3x^4 + 1$$

$$(g(x))^2 = g(x) \cdot g(x) = (3x^2+1)(3x^2+1) = (3x^2+1)^2$$

$$= 9x^4 + 6x^2 + 1$$
